Glucose Optimal Control System in Type I Diabetes Treatment: Design and Simulation

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Optimal Control Course

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Introduction

- Problem: automatic glucose level control in diabetes treatment.
- Other techniques have been tested over time:
  - PID controllers (see [5])
  - PD controllers with pole assignment strategy (see [6])
  - MPC with self-tuning controller (see [7] and [8]).
- Topic: optimal control system based on the linear quadratic control (LQC) problem theory for insulin delivery in a type I diabetic patient.
- Servocontrol and Regulatory approach.
  - Test on Ackerman's linearized model.
  - Test on nonlinear Bergman's minimal model.

Mathematical Model

Ackerman's Model

- Nonlinear interaction between glucose and insulin:

$$\dot{G}(t) = f_1(G, H) + p(t)$$
$$\dot{H}(t) = f_2(G, H) + u(t)$$

- Linearized model:

$$\begin{bmatrix} \dot{g}(t) \\ \dot{h}(t) \end{bmatrix} = \begin{bmatrix} -m_1 & -m_2 \\ 0 & -m_3 \end{bmatrix} \begin{bmatrix} g(t) \\ h(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \quad \begin{bmatrix} g(0) \\ h(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} g(t) \\ h(t) \end{bmatrix}$$

- $G(t)$: glucose concentration value (mg/dL), $\mathrm{H}(t)$: insulin concentration value (mU/L)
- $\mathrm{g}(t) = G(t) - G_0$, $h(t) = H(t) - H_0$
- $m_1 = 0.0009$ $m_2 = 0.0031$, $m_3 = 0.0415$ (given by experimental data [9])

Mathematical Model

Bergman's Minimal Model

- Bergman's minimal model is defined by the following equations:

$$\dot{G}(t) = -P_1 G(t) - X(t)[G(t) + G_b] + d(t)$$
$$\dot{X}(t) = -P_2 X(t) + P_3 I(t)$$
$$\dot{I}(t) = -n[I(t) + I_b] + \frac{1}{V_1} u(t)$$

- $G(t)$: glucose concentration value (mg/dL) (Gb basal glucose value);
- $I(t)$: insulin concentration value (mU/L) (Ib basal insulin value);
- $X(t)$: proportional to the insulin concentration in the remote compartment;
- $d(t)$: external glucose disturbance;
- $u(t)$: exogenous insulin.

Controller

Servocontrol Approach

- Minimizing the objective function

$$J(u) = \int_0^\infty [\eta(x_1(t) - x_d)^2 + \rho u^2(t)]\mathrm{d}t \qquad \textbf{(i)}$$

- Solution: given by

$$u(t) = -R^{-1}B^{\mathrm{T}}(Px(t) + \mu)$$

with P solution of the Riccati equation

$$PA + A^{\mathrm{T}}P - PBR^{-1}B^{\mathrm{T}}P + Q = 0 \qquad \textbf{(ii)}$$

and μ retrieved by

$$(PA + A^{\mathrm{T}}P - PBR^{-1}B^{\mathrm{T}}P + Q)x(t) = (PBR^{-1}B^{\mathrm{T}} - A^{\mathrm{T}})\mu + Qx_D \qquad \textbf{(iii)}$$

solving

$$\mu = -(PBR^{-1}B^{\mathrm{T}} - A^{\mathrm{T}})^{-1}Qx_D$$

Controller

LQC problem: how to get (ii) and (iii)

- The (i) is equivalent to the general cost function of the LQC problem

$$J(u) = \frac{1}{2}e^{\mathrm{T}}(t_f)Se(t_f) + \frac{1}{2}\int_{t_0}^{t_f}[e^{\mathrm{T}}(t)Q(t)e(t) + u^{\mathrm{T}}(t)R(t)u(t)]\mathrm{d}t$$

when $S = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$, $\mathrm{Q} = \begin{bmatrix}2\eta & 0\\ 0 & 0\end{bmatrix}$, $R = 2\rho$, $t_0 = 0$, $t_f = \infty$.

- With optimal control law

$$u(t) = -R(t)^{-1}B(t)^{\mathrm{T}}\lambda(t)$$

where the costate is

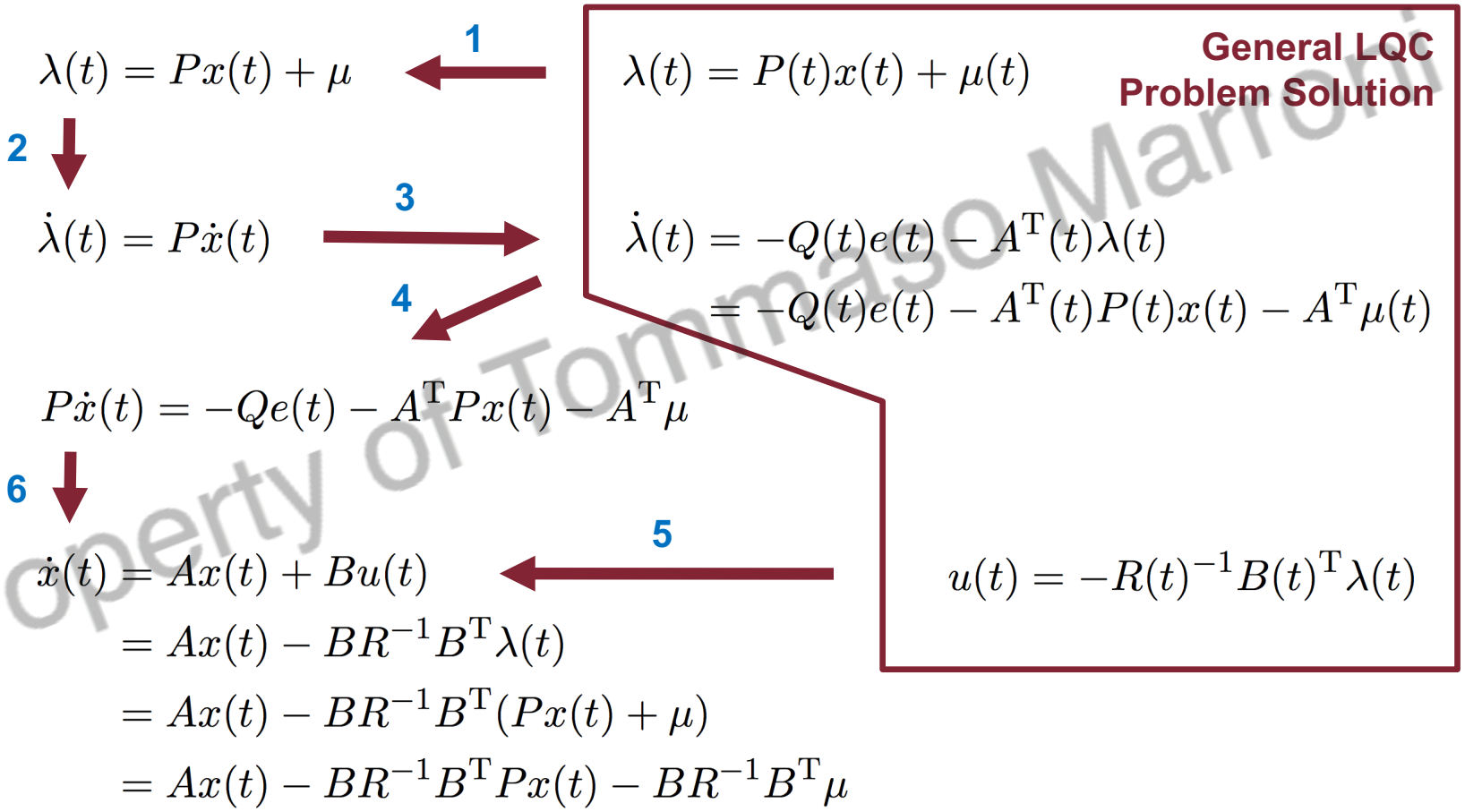
$$\lambda(t) = P(t)x(t) + \mu(t)$$

with dynamics

$$\begin{aligned}\dot{\lambda}(t) &= -Q(t)e(t) - A^{\mathrm{T}}(t)\lambda(t)\\ &= -Q(t)e(t) - A^{\mathrm{T}}(t)P(t)x(t) - A^{\mathrm{T}}\mu(t)\end{aligned}$$

Controller

LQC problem: how to get (ii) and (iii)



Controller

Regulatory Approach

- Minimizing the objective function

$$J(u) = \int_0^\infty [\eta x_1^2(t) + \rho u^2(t)]\mathrm{d}t$$

- Solution:

$$u(t) = -R^{-1}B^{\mathrm{T}}Px(t)$$

with P, again, solution of the Riccati equation:

$$PA + A^{\mathrm{T}}P - PBR^{-1}B^{\mathrm{T}}P + Q = 0$$

Simulation

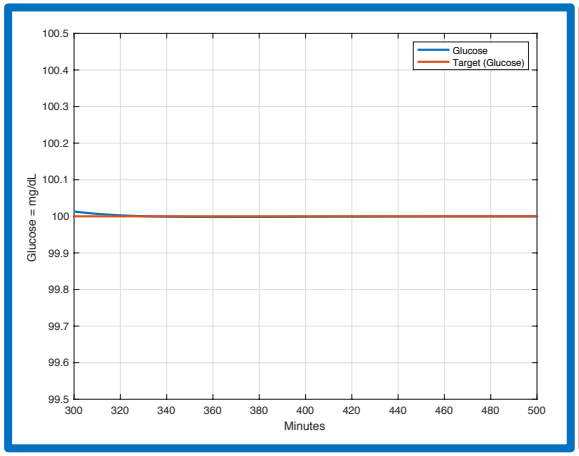
Simulation Parameters

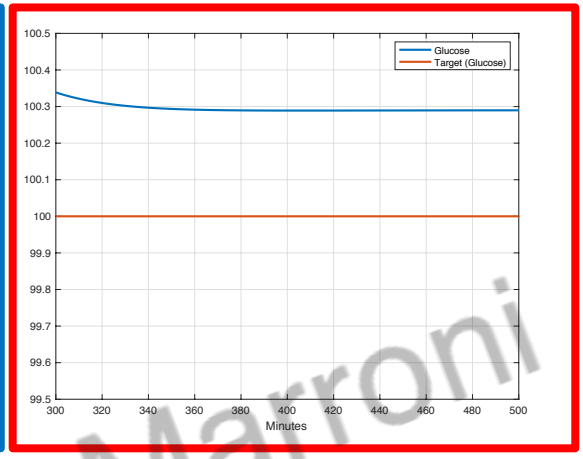
	Model Parameters	Simulation Parameters
Ackerman's Model	• $m_1 = 0.0009$ • $m_2 = 0.0031$ • $m_3 = 0.0415$ From [9]	• $K_e = \begin{bmatrix} 0.2021 \\ -2.1023 \end{bmatrix}$ • $\eta = 1$ • $\rho = 10$
Bergman's Model	• $P_1 = 0$ • $P_2 = 0.025$ • $P_3 = 0.000013$ • $V_1 = 12$ • $G_b = 81$ • $I_b = 15$ • $n = 0.09$ From [3]	• $\eta = 1$ • $\rho = 40$

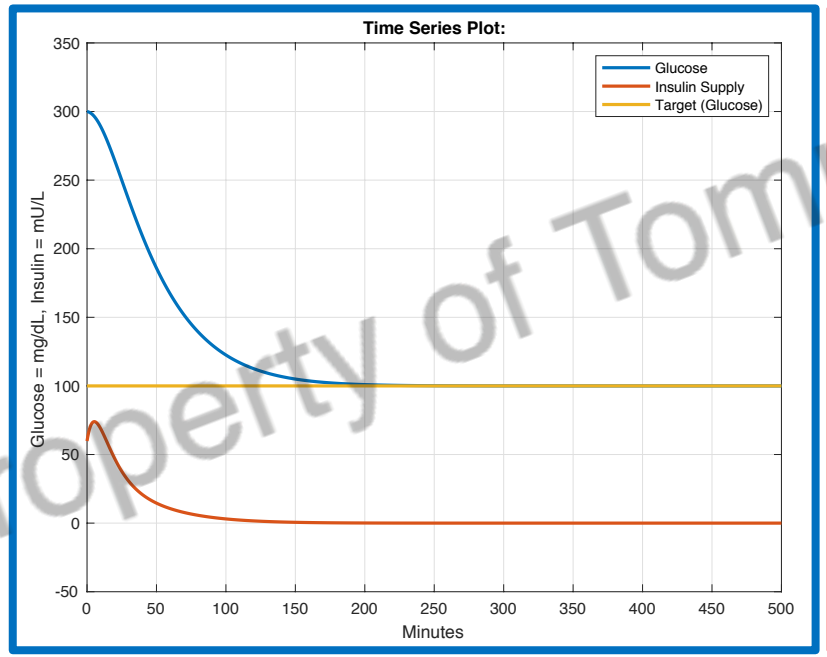
Simulation

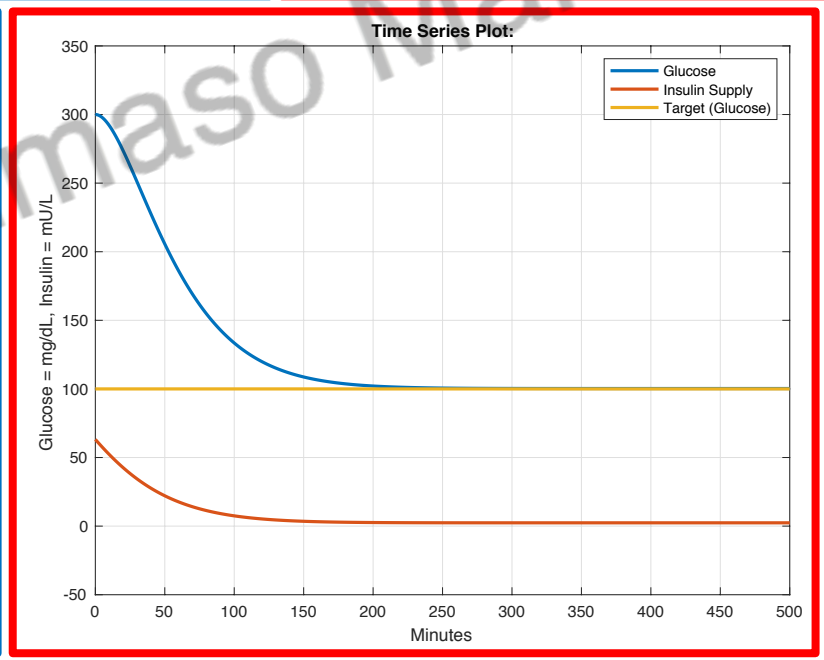
Ackerman's Model

- **Regulatory**
- **Servocontrol**





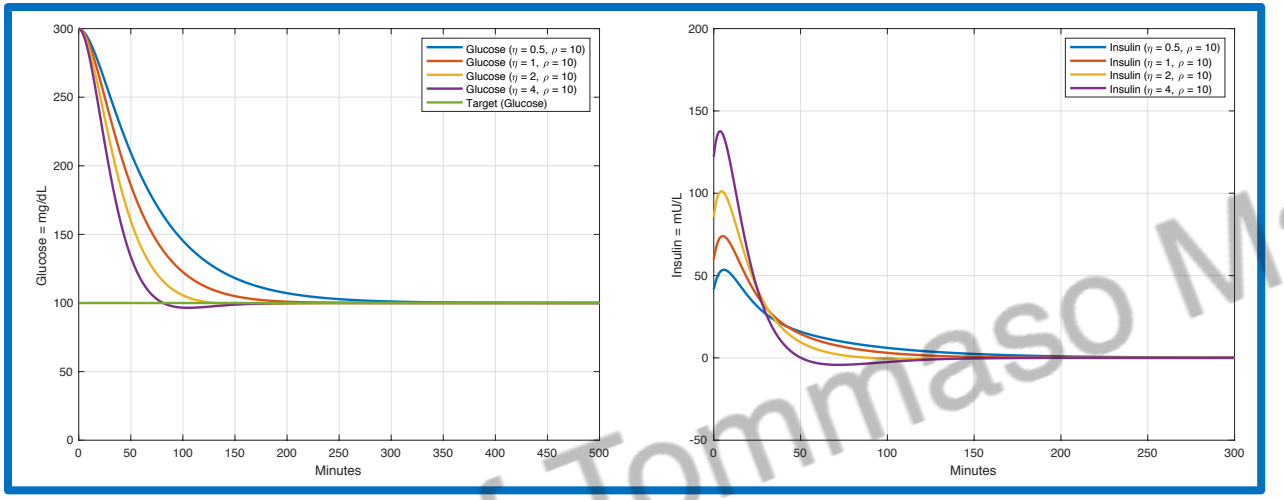


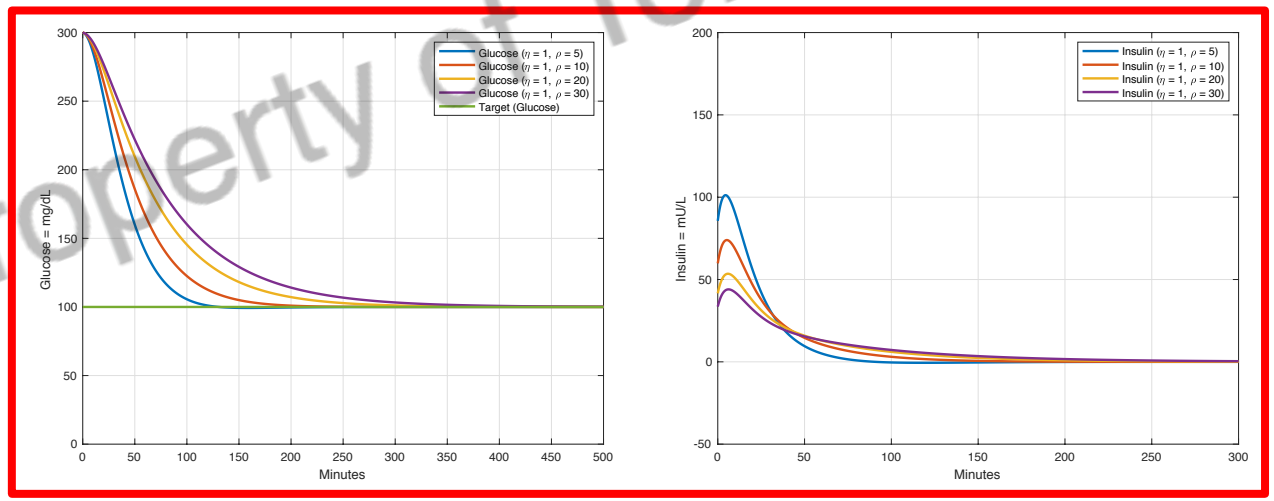


- State observer: $\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + K_e[y(t) - C\hat{x}(t)]$

Simulation

Ackerman's Model





Changing η

Changing ρ

Simulation

Bergman's Minimal Model

- Bergman's Minimal Model:

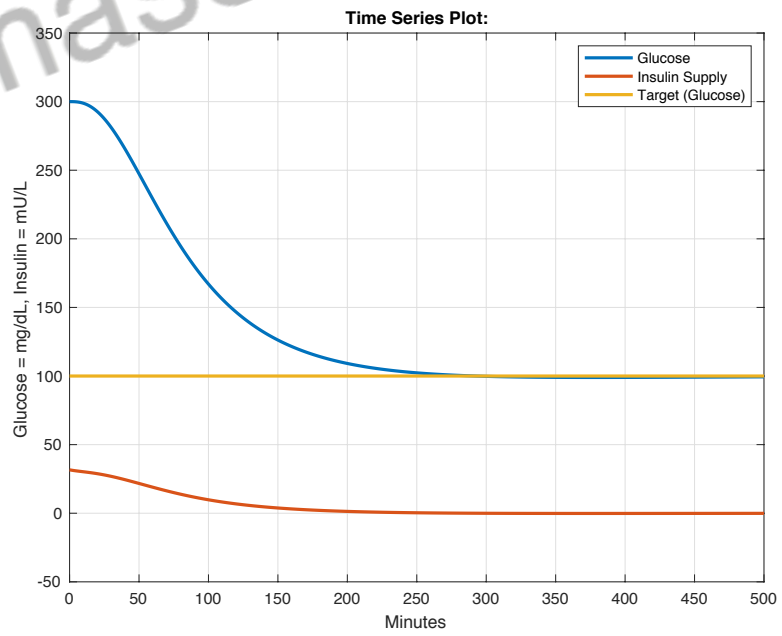
$$\dot{G}(t) = -P_1 G(t) - X(t)[G(t) + G_b] + d(t)$$
$$\dot{X}(t) = -P_2 X(t) + P_3 I(t)$$
$$\dot{I}(t) = -n[I(t) + I_b] + \frac{1}{V_1} u(t)$$

- Linearized Model

$$\Delta\dot{G}(t) = -P_1 \Delta G(t) - \frac{P_3}{P_2} G_b \Delta I(t)$$
$$\Delta\dot{I}(t) = -n\Delta I(t) + \frac{1}{V_1}\Delta u(t)$$

with matrices

$$A = \begin{bmatrix} -P_1 & \frac{P_3}{P_2} G_b \\ 0 & -n \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{V_1} \end{bmatrix}$$



Conclusion

- In this work we provided and tested an optimal control solution to the problem of glucose level regulation in type I diabetic patient.
- We could test robustness using a Monte Carlo simulation (see [1]).
  - However, the LQR build on the linear approximation of the Bergman's model works pretty well on the original nonlinear system, this already shows a certain degree of robustness.
- Choose carefully the weighting factors. Glucose oscillation under the equilibrium value are synonymous of hypoglycemia.
- This control law does not contain a feedforward term which is actually present in the traditional open-loop treatment.
- This control system does not consider the sensor and actuator models.
- It requires continuous glucose measurement and this may not be easily achievable (also the comfort of the patient should be always considered).

References

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- [4] Carlo Bruni, Gianni Di Pillo, Metodi variazionali per il controllo ottimo, Aracne, Ottobre 2015, 978-88-548-1380-9.
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