Suite of Feedback Controllers for Unicycle Robots

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Autonomous and Mobile Robotics (AMR)
Final Project (FP7)

Summary

Introduction

Goal of the project

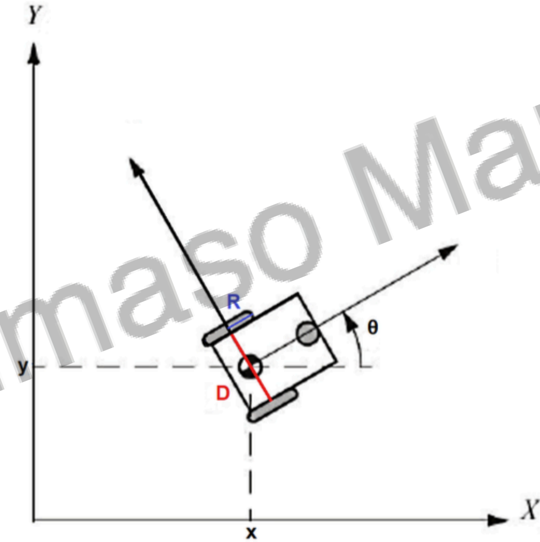
- The goal of the project is the control of a **differential-drive mobile robot** using a set of different types of control schemes.
- All controllers are implemented and simulated in Simulink.
- Controllers: 3 for trajectory tracking, 3 for posture stabilization.

Introduction

Model of the differential-drive mobile robot

The following equations describe the state-space representation for the chosen robot:

$$\begin{cases} \dot{x} = v\cos\theta \\ \dot{y} = v\sin\theta \\ \dot{\theta} = \omega \end{cases}$$



$$v = \frac{R(\omega_r + \omega_l)}{2}$$
$$\omega = \frac{R(\omega_r - \omega_l)}{D}$$

$$\rightarrow \begin{cases} \dot{x} = \frac{R}{2}\cos\theta\omega_r + \frac{R}{2}\cos\theta\omega_l \\ \dot{y} = \frac{R}{2}\sin\theta\omega_r + \frac{R}{2}\sin\theta\omega_l \\ \dot{\theta} = R\frac{\omega_r}{D} - R\frac{\omega_l}{D} \end{cases}$$

Introduction

Control problems

The controllers taken into consideration are used to solve either one of these problems:

Tracking problem: Make the robot follow a given reference trajectory $(x_d(t), y_d(t))$

Regulation problem: Make the robot reach a desired state configuration (x_f, y_f, θ_f) starting from a given initial one (x_i, y_i, θ_i)

Introduction

Localization

Computing the correct control actions requires the knowledge of the position and the orientation of the robot in order to use a feedback loop. In this work, two ways of doing so are used:

GPS: An external system sends information to the robot about its configuration.

Odometry: Information is obtained by numerical integration of the kinematic model over a sampling interval (using **2° order Runge-Kutta** algorithm**)**.

$$x_{k+1} = x_k + v_k T_s \cos\left(\theta_k + \frac{\omega_k T_s}{2}\right)$$

$$y_{k+1} = y_k + v_k T_s \sin\left(\theta_k + \frac{\omega_k T_s}{2}\right)$$

$$\theta_{k+1} = \theta_k + \omega_k T_s$$

Trajectory Tracking

- Linear Approximation

- RAS Controllers
  - taken from [1]

- CST Controller
  - taken from [2]

[1] Luigi Freda, Giuseppe Oriolo, *Vision-based interception of a moving target with a nonholonomic mobile robot*, Elsevier, Robotics and Autonomous Systems 55 (2007) 419-432.

[2] Giuseppe Oriolo, Alessandro De Luca, *WMR Control Via Dynamic Feed- back Linearization: Design, Implementation, and Experimental Validation*, IEEE Transactions on Control Systems Technology, Vol. 10, No. 6, 835-852, November 2002.

Linear Approximation

- *Linearize the error dynamics around the reference trajectory*
- Control law:

$$v = v_d \cos e_3 - k_1 e_1$$
$$\omega = \omega_d + k_2 e_2 + k_3 e_3$$

- e_1, e_2 and e_3 (rotated component of the error):

$$e = \begin{pmatrix} e_1 \\ e_2 \\ e_3 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_d - x \\ y_d - y \\ \theta_d - \theta \end{pmatrix}$$

Linear Approximation

- k_1, k_2 as follows:

$$k_1 = k_3 = 2\zeta a$$
$$k_2 = \frac{a^2 - \omega_d^2}{v_d}$$

- v_d and ω_d are obtained from the desired trajectory using flatness:

$$\theta_d = \mathrm{atan2}(\dot{\mathrm{y}}_\mathrm{d}, \dot{\mathrm{x}}_\mathrm{d}) + \mathrm{k}\pi$$
$$v_d = \pm\sqrt{\dot{x_d}^2 + \dot{y_d}^2}$$
$$\omega_d = \frac{\ddot{y_d}\dot{x_d} - \ddot{x_d}\dot{y_d}}{\dot{x_d}^2 + \dot{y_d}^2}$$

- Asymptotic stability is **not guaranteed**, unless v_d and ω_d are constant.
  - even for these two types of references, the asymptotic stability is **not global**.

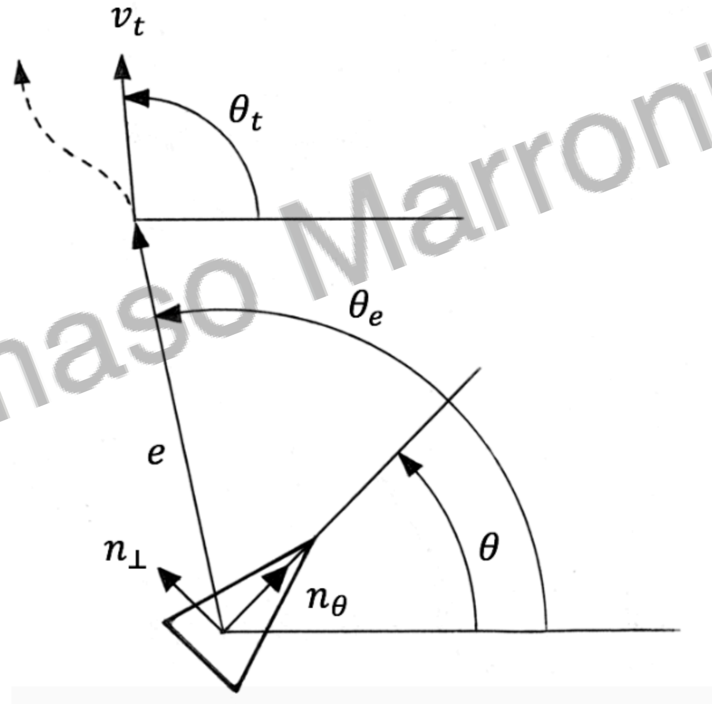
RAS

- First controller (RAS 1):

$$v = k_1 e^T n_\theta + v_t^T n_\theta$$
$$\omega = k_2(\theta_e - \theta) + \dot{\theta}_e$$

- Second controller (RAS 2)

$$v = k_1 e^T n_\theta + v_t^T n_\theta$$
$$\omega = k_2(\theta_e - \theta) + k_3 e^T n_\perp + \dot{\theta}_e$$



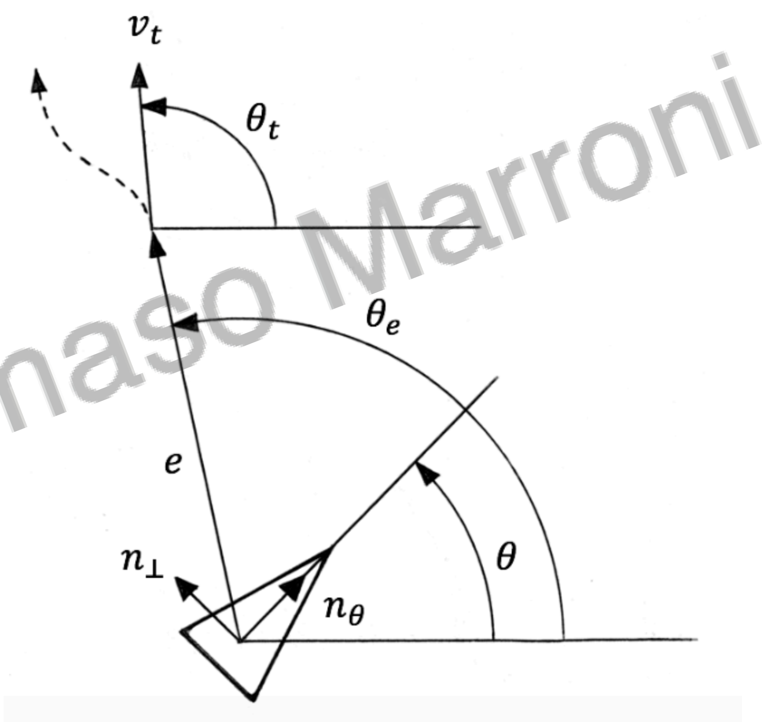
RAS

- Main difference from [1]: usage of a given reference instead of a system composed of a camera and a moving target.

- Presence of component depending on e in ω of RAS 2.

  → faster alignment

- Unicycle can follow only trajectories described by $x_d(t)$, $y_d(t)$ of class $\bar{C}^2$.



CST

- *Nonlinear controller based on Dynamic Feedback Linearization*.
- Dynamic compensator:

$$\dot{\xi} = u_1 \cos\theta + u_2 \sin\theta$$
$$v = \xi$$
$$\omega = \frac{u_2 \cos\theta - u_1 \sin\theta}{\xi}$$

- The feedback (providing exponential stabilization):

$$u_1 = \ddot{x_d} + k_{p1}(x_d - x) + k_{d1}(\dot{x_d} - \dot{x})$$
$$u_2 = \ddot{y_d} + k_{p2}(y_d - y) + k_{d2}(\dot{y_d} - \dot{y})$$

- $\xi = 0$ is a singularity for the dynamic feedback compensator.

Regulation

- *Drive the unicycle to a desired configuration* q_d.
- Tracking a trajectory which stops in q_d wouldn't work:

  – Error-dynamics-based controllers require persistent trajectories.

  – Need a point B as output when performing I/O linearization.

I/0 Linearization leads point B to destination

Regulation

- Cartesian Regulation
- Posture Regulation
- CST Regulation
  - taken from [2]

[2] Giuseppe Oriolo, Alessandro De Luca, *WMR Control Via Dynamic Feed- back Linearization: Design, Implementation, and Experimental Validation*, IEEE Transactions on Control Systems Technology, Vol. 10, No. 6, 835-852, November 2002.

Cartesian Regulation

- *Drive the unicycle to the desired Cartesian position* (x_d, y_d)

- Model of the control:

$$v = k_1 \boldsymbol{e_p}^T \boldsymbol{n}$$
$$\omega = k_2(\mathrm{atan2}(y_e, x_e) - \theta)$$

- With this control law robot reaches the target configuration through a forward motion.

Posture Regulation

- *Drive the unicycle to the* desired configuration (x_d, y_d, θ_d)
- Model of the control:

$$v = k_1 \rho \cos\gamma$$

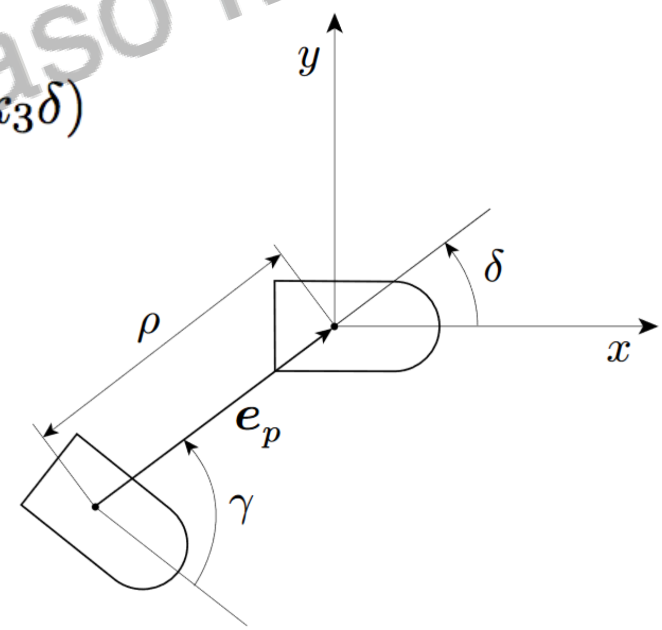
$$\omega = k_2\gamma + k_1 \frac{\sin\gamma\cos\gamma}{\gamma}(\gamma + k_3\delta)$$

- Polar coordinates:

$$\rho = \sqrt{x^2 + y^2}$$

$$\gamma = \mathrm{atan2(y, x)} - \theta + \pi$$

$$\delta = \gamma + \theta$$



CST Regulation

- Solve posture stabilization problem
- Model of the control:
  - Compensator:

$$\dot{\xi} = u_1 \cos\theta + u_2 \sin\theta$$
$$v = \xi$$
$$\omega = \frac{u_2 \cos\theta - u_1 \sin\theta}{\xi}$$

  - Feedback:

$$u_1 = -k_{p1}x - k_{d1}\dot{x}$$
$$u_2 = -k_{p2}y - k_{d2}\dot{y}$$

CST Regulation

- Exponential convergence to the origin is guaranteed if $q_0 \in Q/Q^*$, with Q^* such that:

$$Q^* = \{q \in Q : (x = 0, \cos\theta \geq 0) \; OR \\ (y = 0, \cos\theta = -1) \; OR \\ (x = y = 0)\}$$

and:

$$k_{d1}^2 - 4k_{p1} = k_{d2}^2 - 4k_{p2} > 0$$

$$k_{d2} - k_{d1} > 2\sqrt{k_{d2}^2 - 4k_{p2}}$$

$$\begin{cases} \xi_0 < 0 \;\; (backward\ motion) & if\ q_0 \in RHP/Q^* \\ \xi_0 > 0 \;\; (forward\ motion) & if\ q_0 \in LHP/Q^* \end{cases}$$

CST Regulation

- Conditions to avoid singularity in $\xi = 0$:

$$\xi_0 \neq 2\frac{k_{p1}x_0 \sin\theta_0 - k_{p2}y_0 \cos\theta_0}{k_{d2} - k_{d1}}$$

- This condition guarantees that eigenvalues characterizing the transient are real.

- When $q_0 \in Q^*$ the controller brings robot to the wrong orientation.

  → a via point is needed

  → global exponential stability.

Simulations

Trajectories for tracking

The following trajectories were used to test the efficiency of the tracking controllers:

- **Linear**:

$$\begin{cases} x_d(t) = t \\ y_d(t) = 0 \end{cases}$$

- **Circular**:

$$\begin{cases} x_d(t) = \cos(t) \\ y_d(t) = \sin(t) \end{cases}$$

- **Linear-Circular succession**:

$$\begin{cases} x_d(t) = t \\ y_d(t) = 0 \end{cases} \quad , t < t_1$$

$$\begin{cases} x_d(t) = t_1 + \cos(t - 4 - \frac{\pi}{2}) \\ y_d(t) = -1 - \sin(t - 4 - \frac{\pi}{2}) \end{cases} \quad , t \geq t_1$$

Simulations

Trajectories for tracking

- **Cubic**:

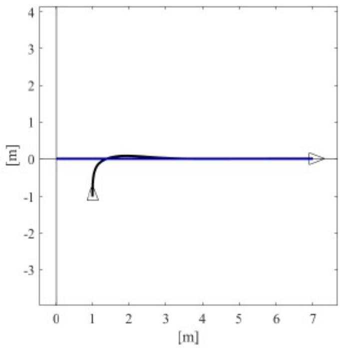
$$\begin{cases} x_d(t) = t \\ y_d(t) = t^3 \end{cases}$$

- **Spiral:**

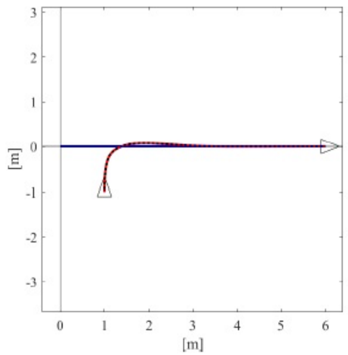
$$\begin{cases} x_d(t) = t\cos t^2 \\ y_d(t) = t\sin t^2 \end{cases}$$

Simulations

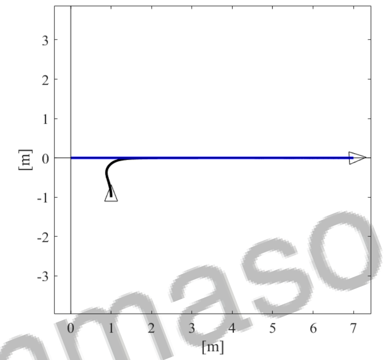
Linear trajectory



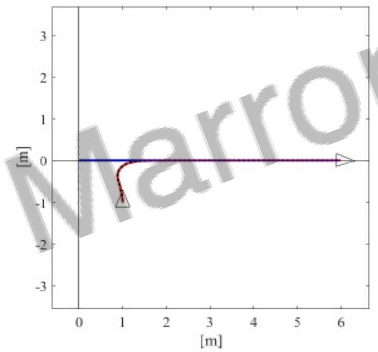
Lin. Approx. GPS



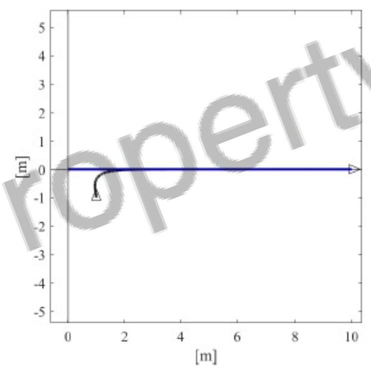
Lin. Approx. R-K



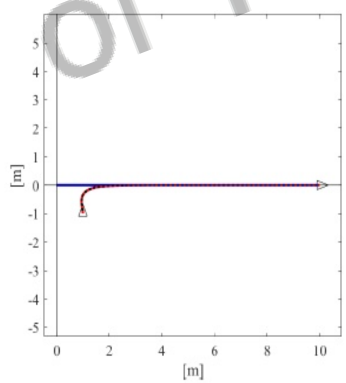
RAS 1 GPS



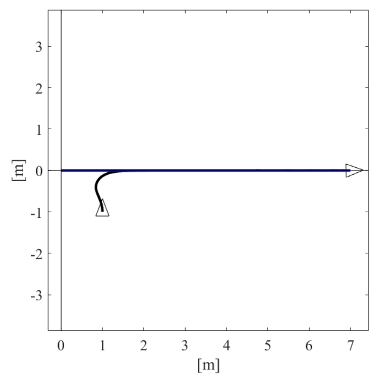
RAS 1 R-K



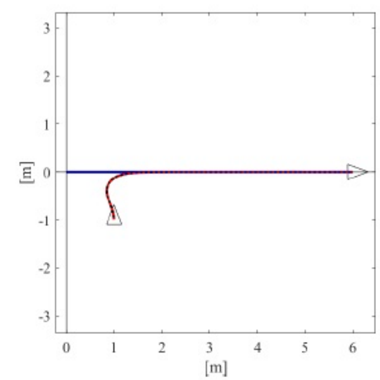
CST GPS



CST R-K



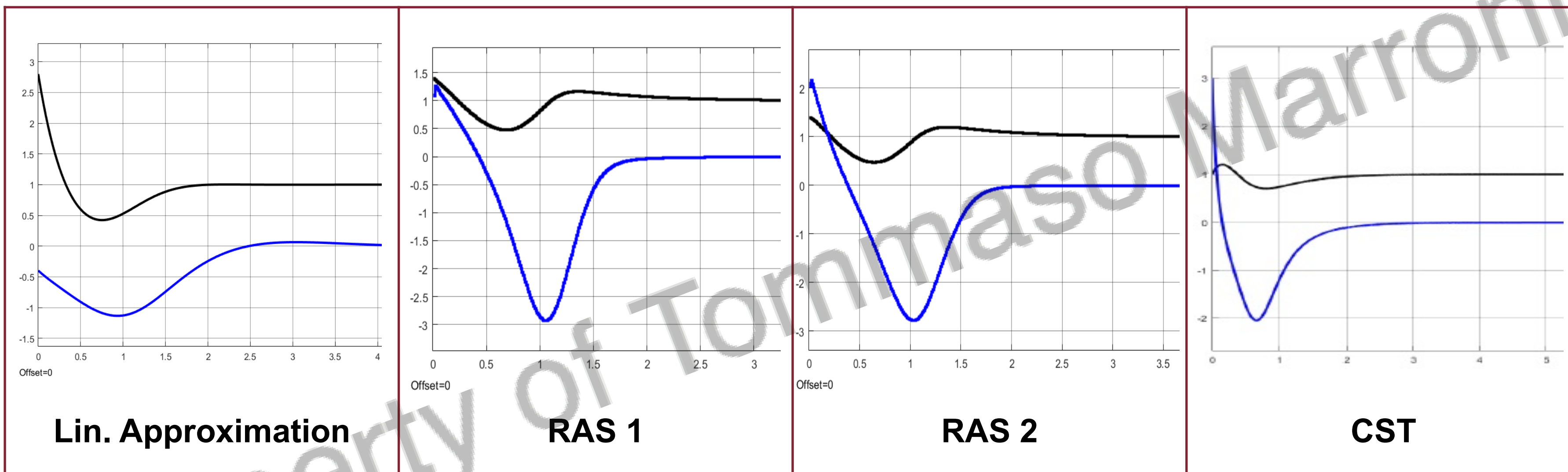
RAS 2 GPS



RAS 2 R-K

Simulations

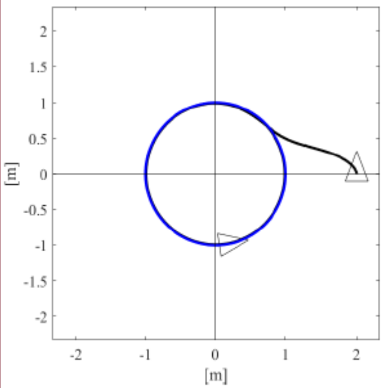
Control actions



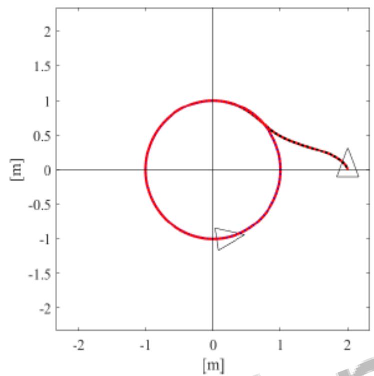
v (black) and ω (blue) in the GPS case.

Simulations

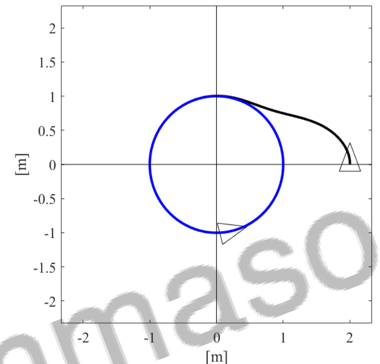
Circular trajectory



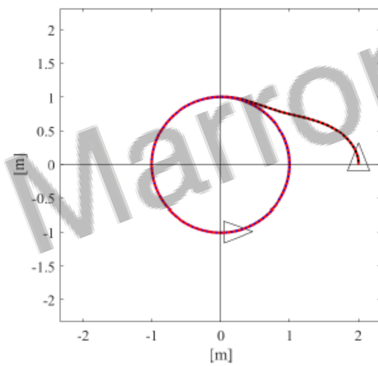
Lin. Approx. GPS



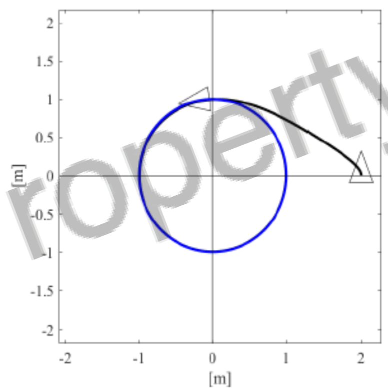
Lin. Approx. R-K



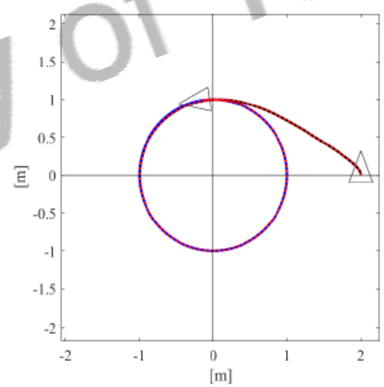
RAS 1 GPS



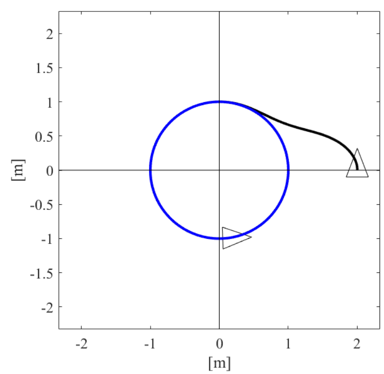
RAS 1 R-K



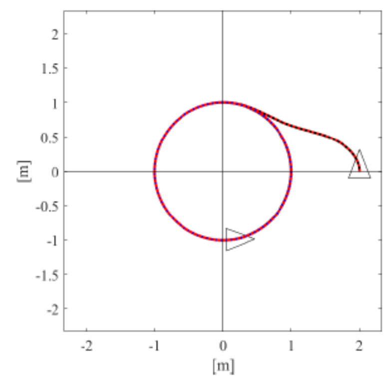
CST GPS



CST R-K



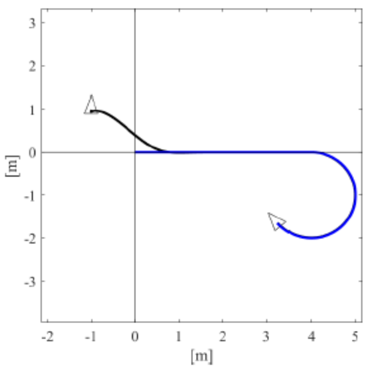
RAS 2 GPS



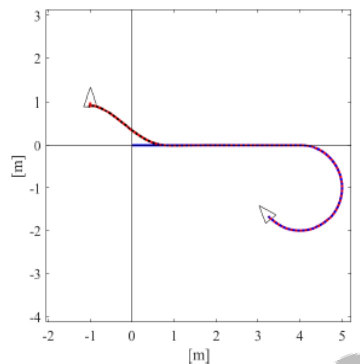
RAS 2 R-K

Simulations

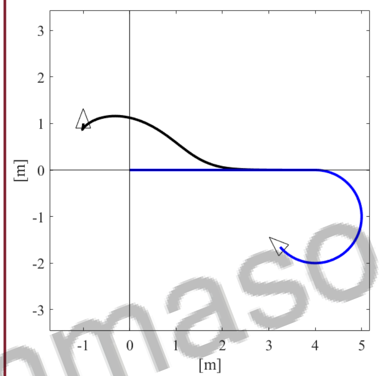
L-C trajectory



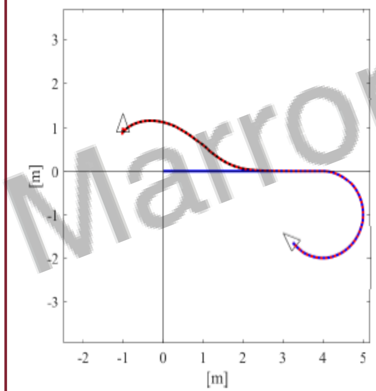
Lin. Approx. GPS



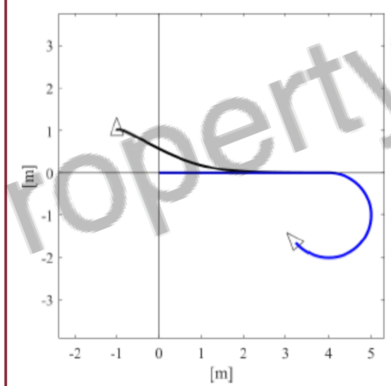
Lin. Approx. R-K



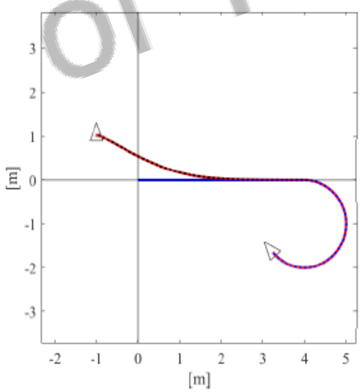
RAS 1 GPS



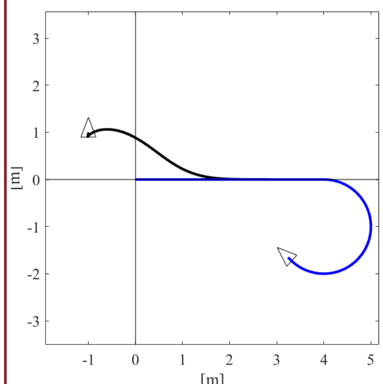
RAS 1 R-K



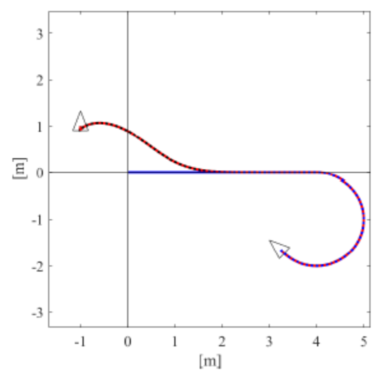
CST GPS



CST R-K



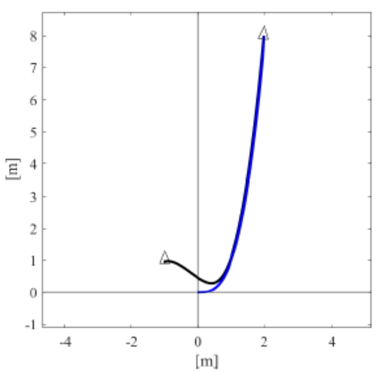
RAS 2 GPS



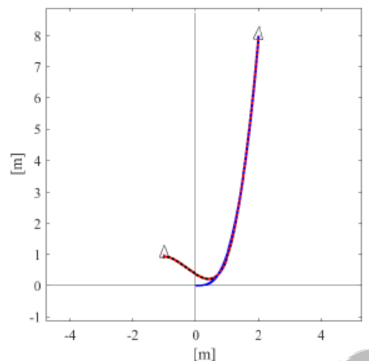
RAS 2 R-K

Simulations

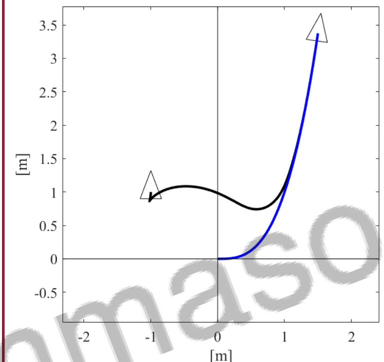
Cubic trajectory



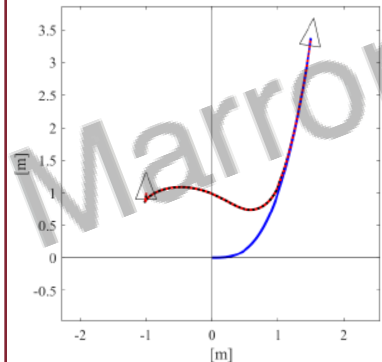
Lin. Approx. GPS



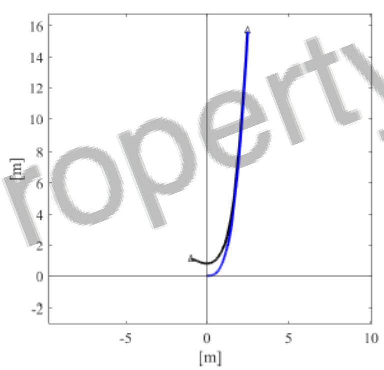
Lin. Approx. R-K



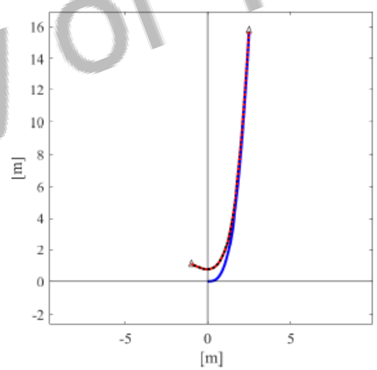
RAS 1 GPS



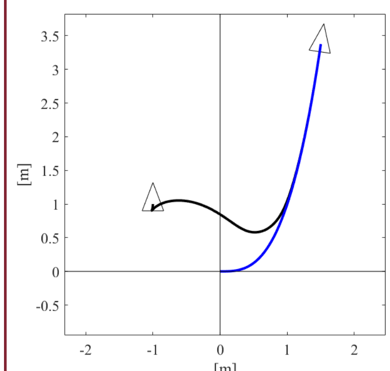
RAS 1 R-K



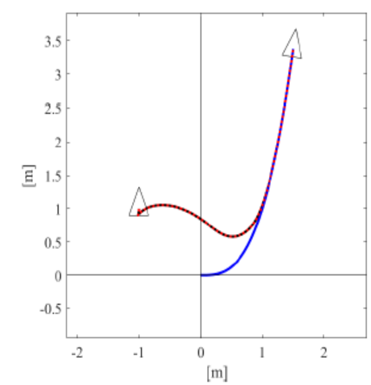
CST GPS



CST R-K



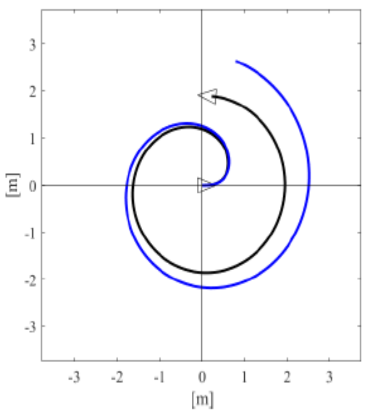
RAS 2 GPS



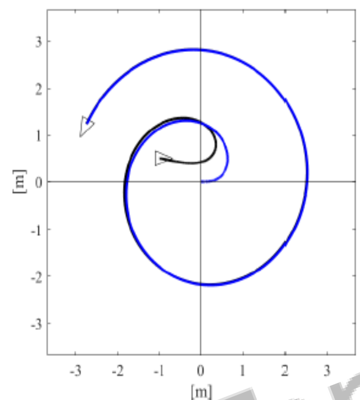
RAS 2 R-K

Simulations

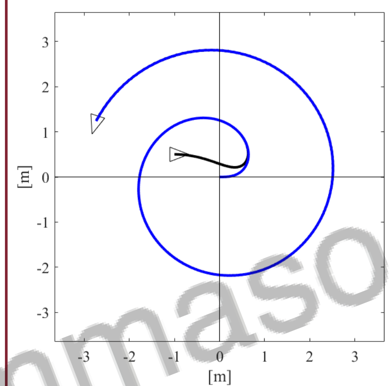
Spiral trajectory



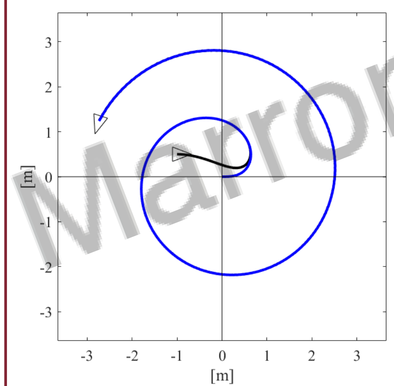
Lin. Approx. GPS



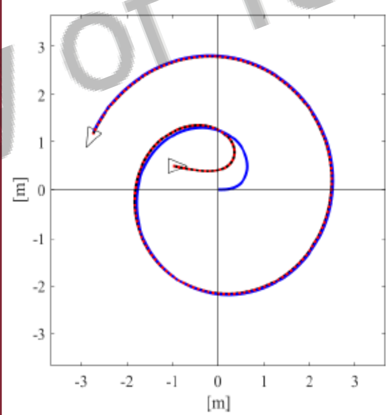
CST GPS



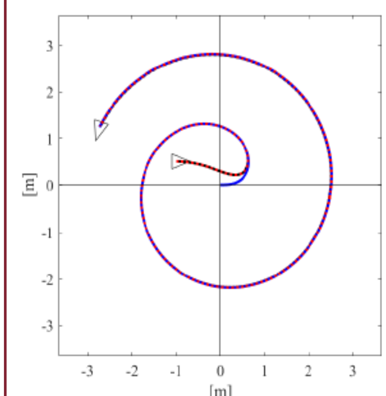
RAS 1 GPS



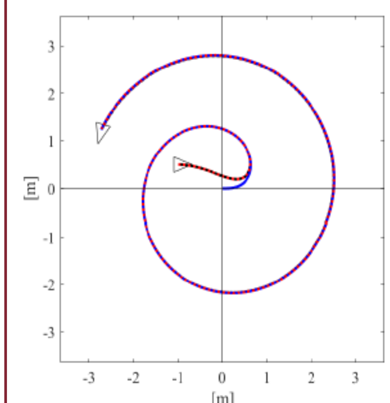
RAS 1 GPS



CST R-K



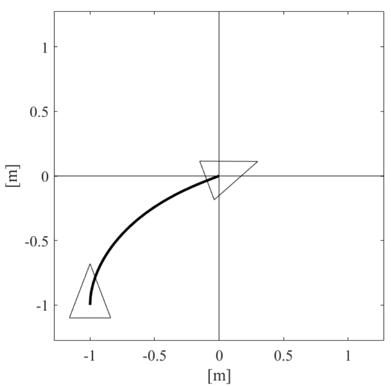
RAS 1 R-K



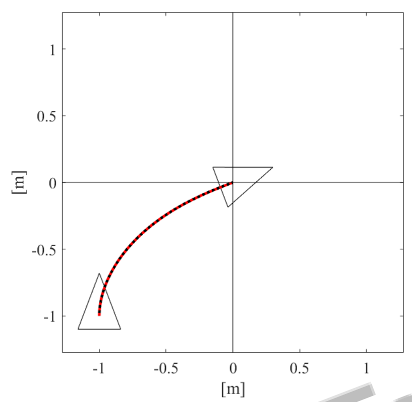
RAS 2 R-K

Simulations

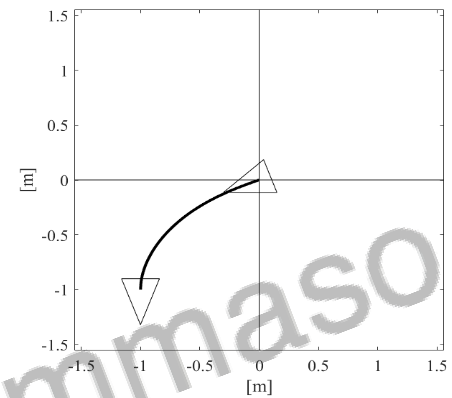
Cartesian Regulation



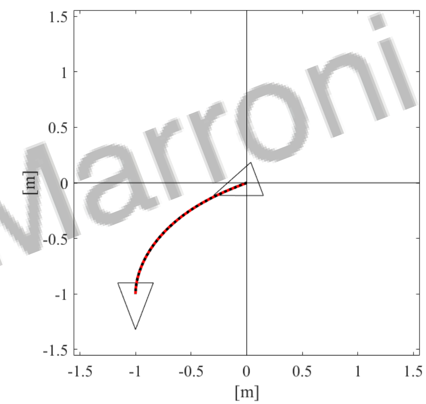
Cart. Reg. FWD GPS



Cart. Reg. FWD R-K



Cart. Reg. BWD GPS

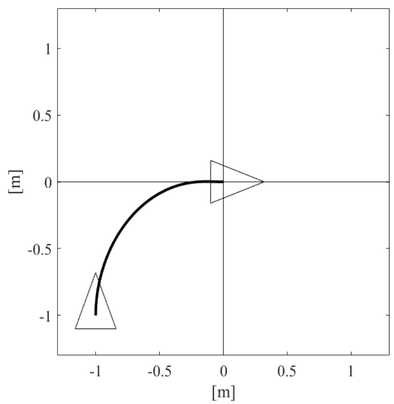


Cart. Reg. BWD R-K

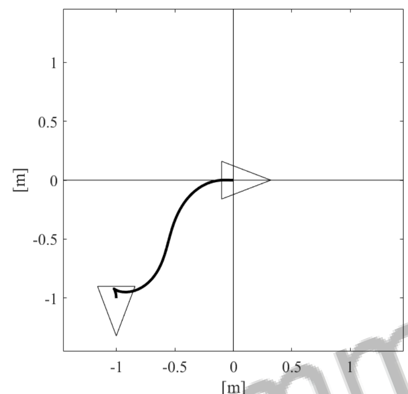
- To avoid useless reversals and reach desired configuration in FWD or BWD motion:
  - Add the term $-\pi sign(atan2(y, x) - \theta + \pi)$ to ω when $e_p^T n \leq 0$.
- To impose only FWD or BWD motion:
  - Multiply v by $sign(e_p^T n)$.
  - Multiply v by $-sign(e_p^T n)$ and add π to ω.

Simulations

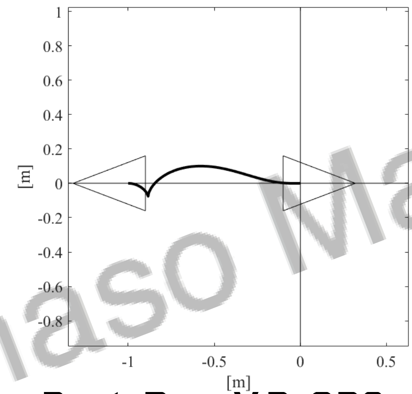
Posture Regulation



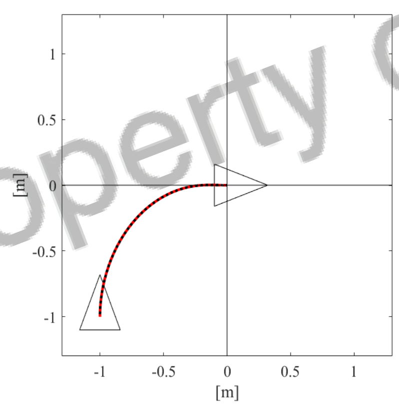
Post. Reg. FWD GPS



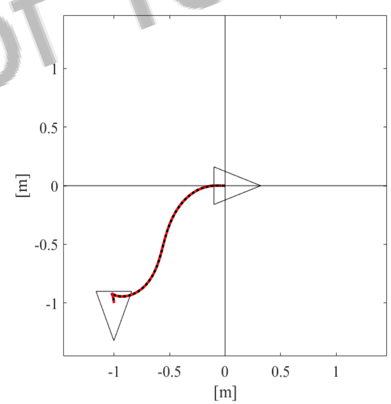
Post. Reg. BWD GPS



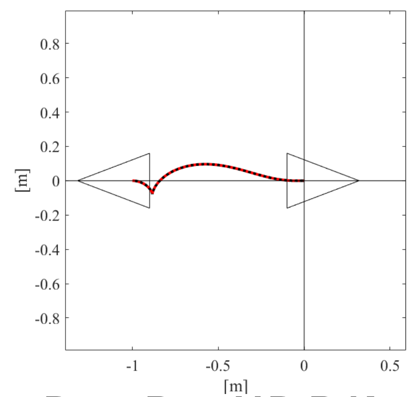
Post. Reg. V.P. GPS



Post. Reg. FWD R-K



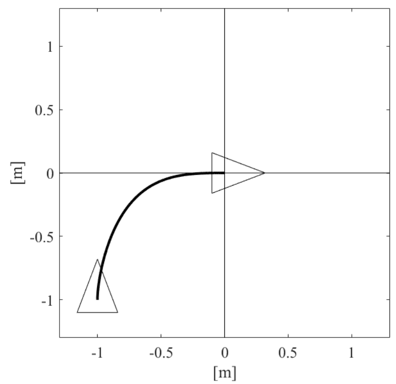
Post. Reg. BWD R-K



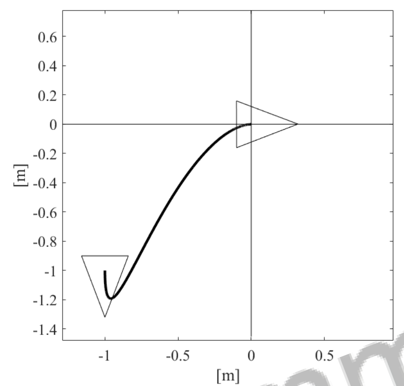
Post. Reg. V.P. R-K

Simulations

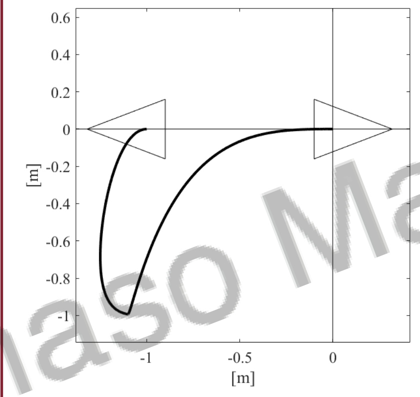
CST Regulation



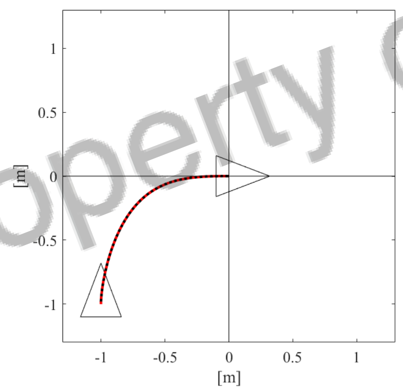
CST Reg. FWD GPS



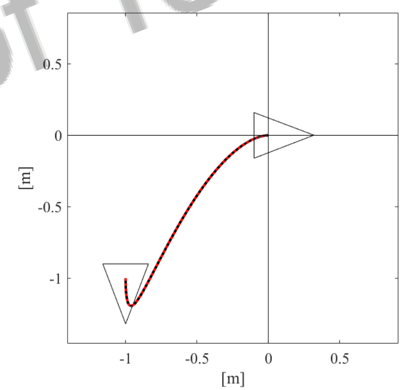
CST Reg. BWD GPS



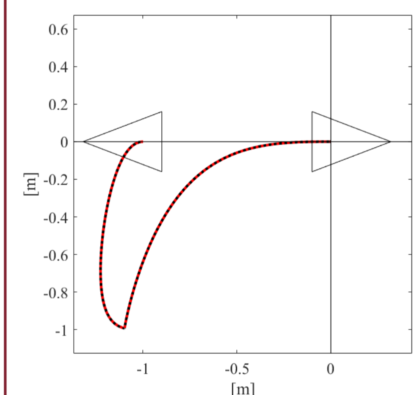
CST Reg. V.P. GPS



CST Reg. FWD R-K



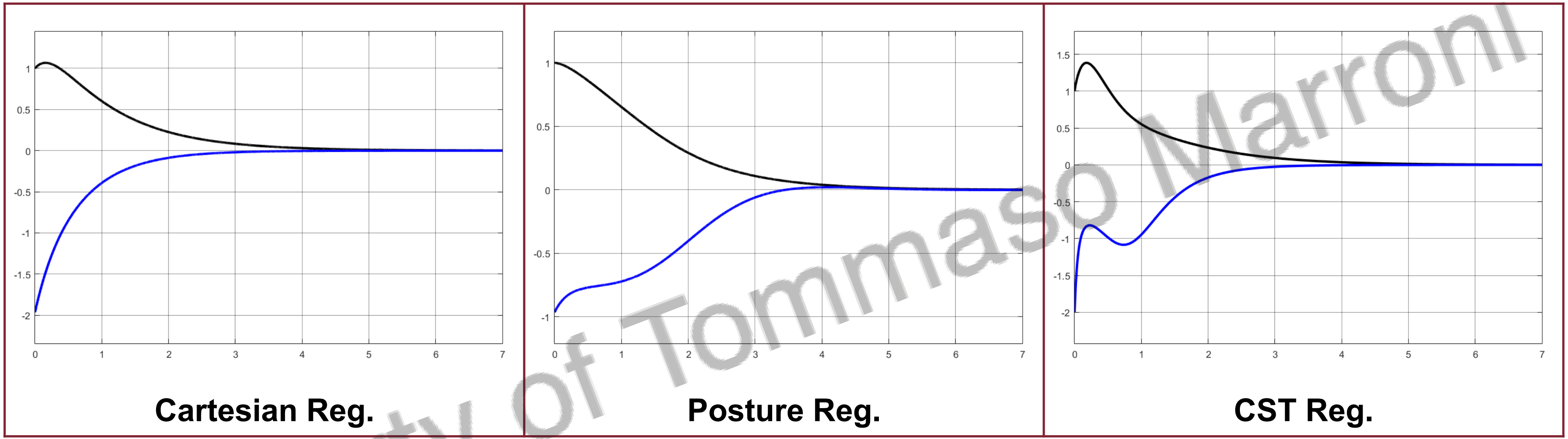
CST Reg. BWD R-K



CST Reg. V.P. R-K

Simulations

Control actions



blue: ω
black: v

Additional Notes

Issue n° 1 [Linear Approximation]

Problem regarding the implementation of $\theta_d - \theta$.

Solution: remove *atan2* and implement a new method to compute angle differences.

- Set $n_d = \begin{pmatrix} \cos\theta_d \\ \sin\theta_d \end{pmatrix}$ and $n = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$.
- $n \cdot n_d = \cos\alpha \quad \Rightarrow \quad \alpha = \arccos(n \cdot n_d)$
- The sign of α is obtained from the z-component of the vectorial product between n and n_d.
- Final equation: $\alpha = sign(n \times n_d)_z \cdot \arccos(n \cdot n_d)$

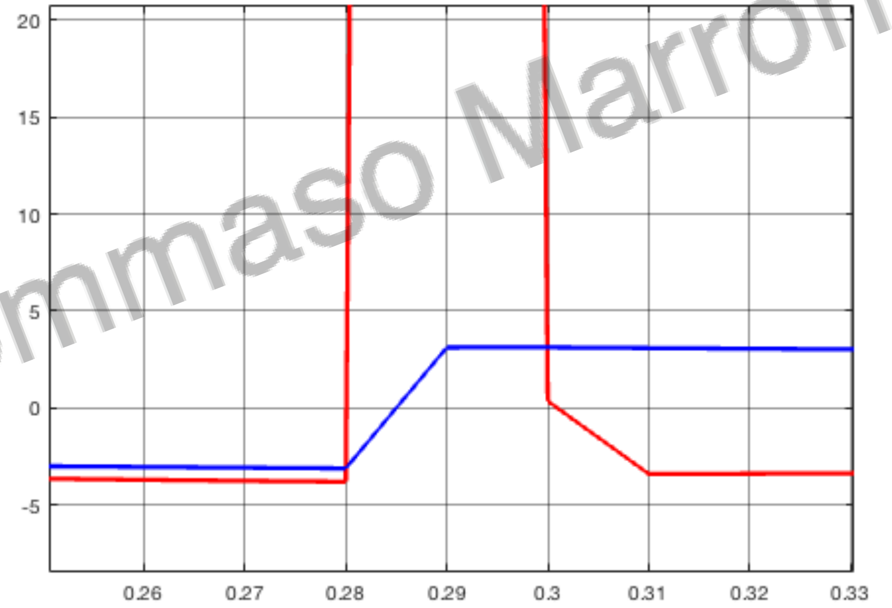
Same problem has been faced with RAS controller ($\theta_e - \theta$, θ_e and θ_t).

Additional Notes

Issue n° 2 [RAS]

Both controllers (RAS 1 and 2) use $\dot{\theta}_e$ and $\dot{\theta}_t$.

<u>Problem</u>: when the angle passes from $-\pi$ to π (or from π to $-\pi$) the derivative grows (decreases) excessively.

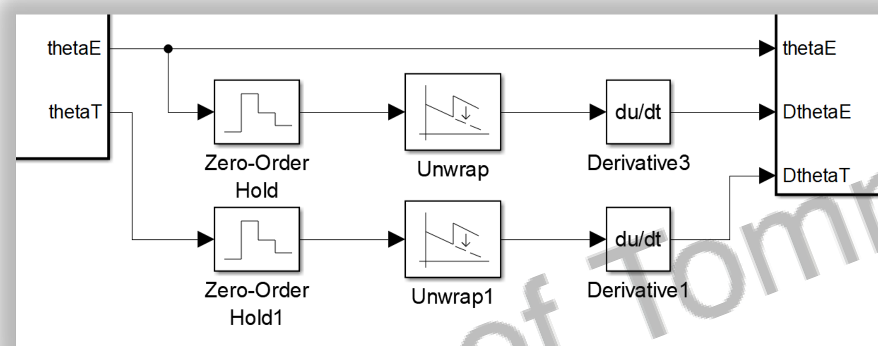


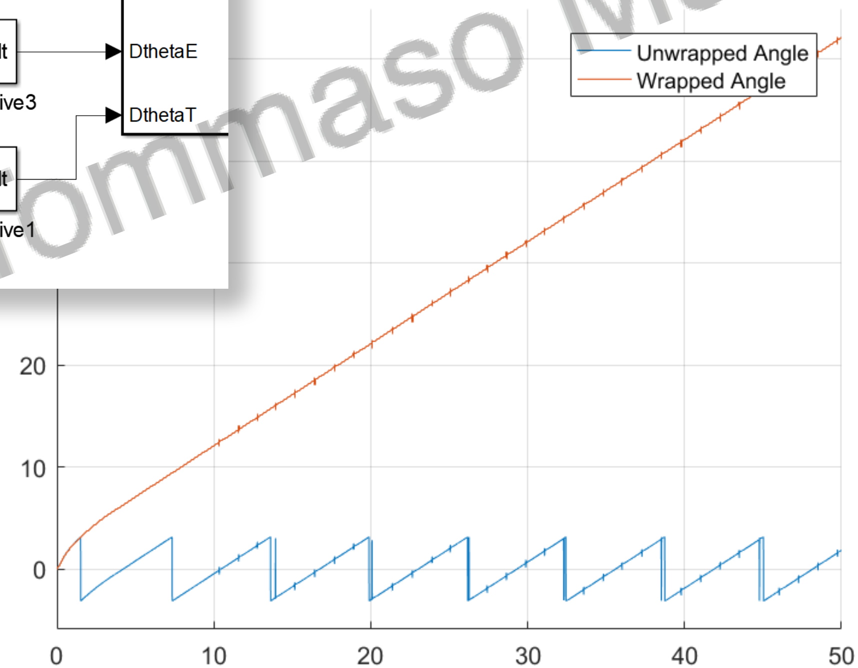
RED: $\dot{\theta}_e$, BLUE: θ_e

Additional Notes

Issue n° 2 [RAS]

<u>Solution</u>: unwrap the angles whose derivative must be computed.





Additional Notes

Issue n° 3 [RAS]

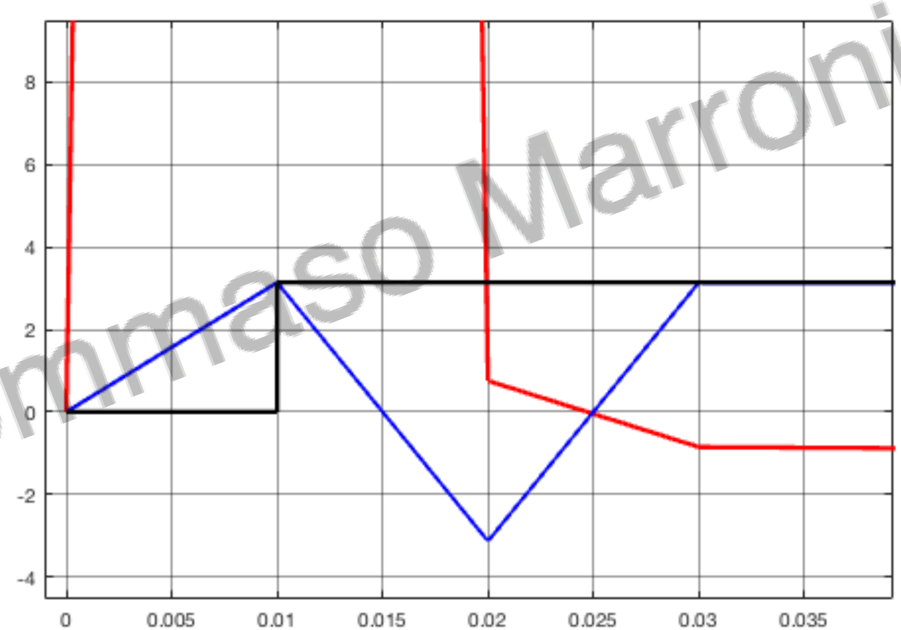
- Problem: ZOH mantains, for the first T_s seconds, the angle θ_e (or θ_t) equal to zero.

↓

Unjustified peak in the derivative term.

↓

Wrong initial control action.



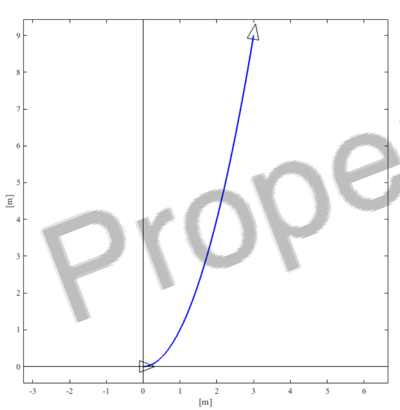
RED: $\dot{\theta}_e$, BLUE: θ_e before unwrap, BLACK: θ_e

- Solution: Neglect the derivative term for the first $2T_s$ seconds.

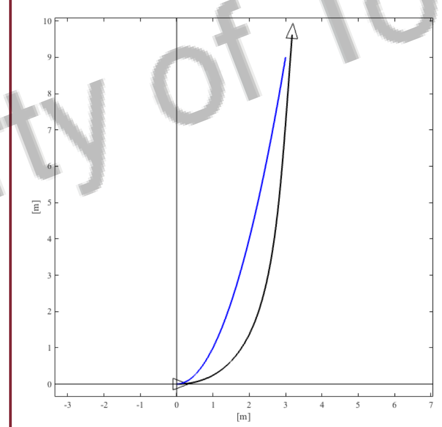
Additional Notes

Issue n° 4 [CST]

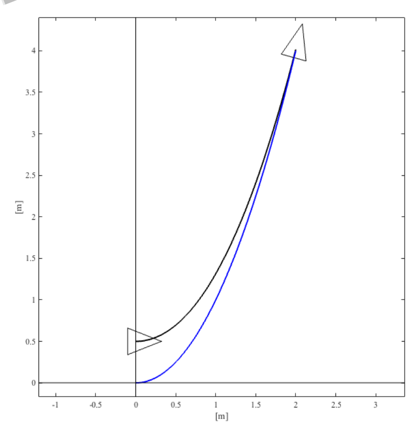
- In general, using <u>derivative blocks</u> in Simulink can lead to complications.
- In particular, in CST:
  - Robot doesn't converge in case of spiral trajectory;
  - Phase shift at steady state in circular trajectory (and steady state error);
  - Longer transients;
  - Less effective control actions;



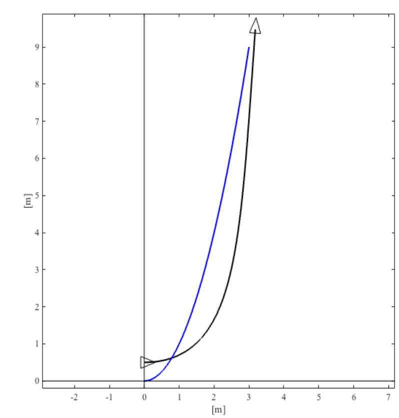
No Deriv. Blocks



Deriv. Blocks

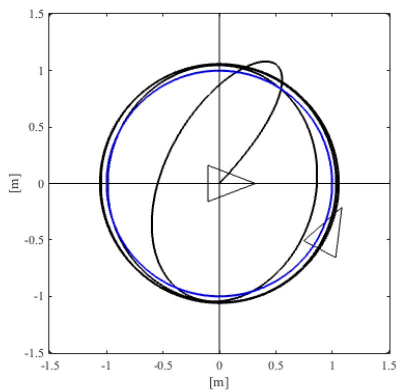


No Deriv. Blocks

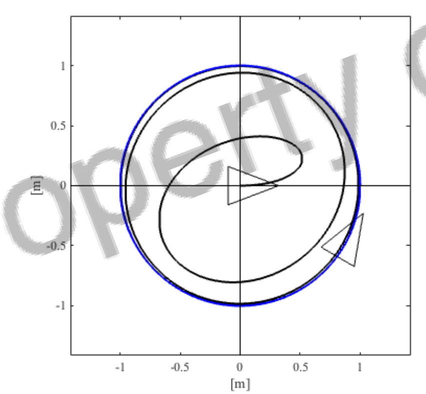


Deriv. Blocks

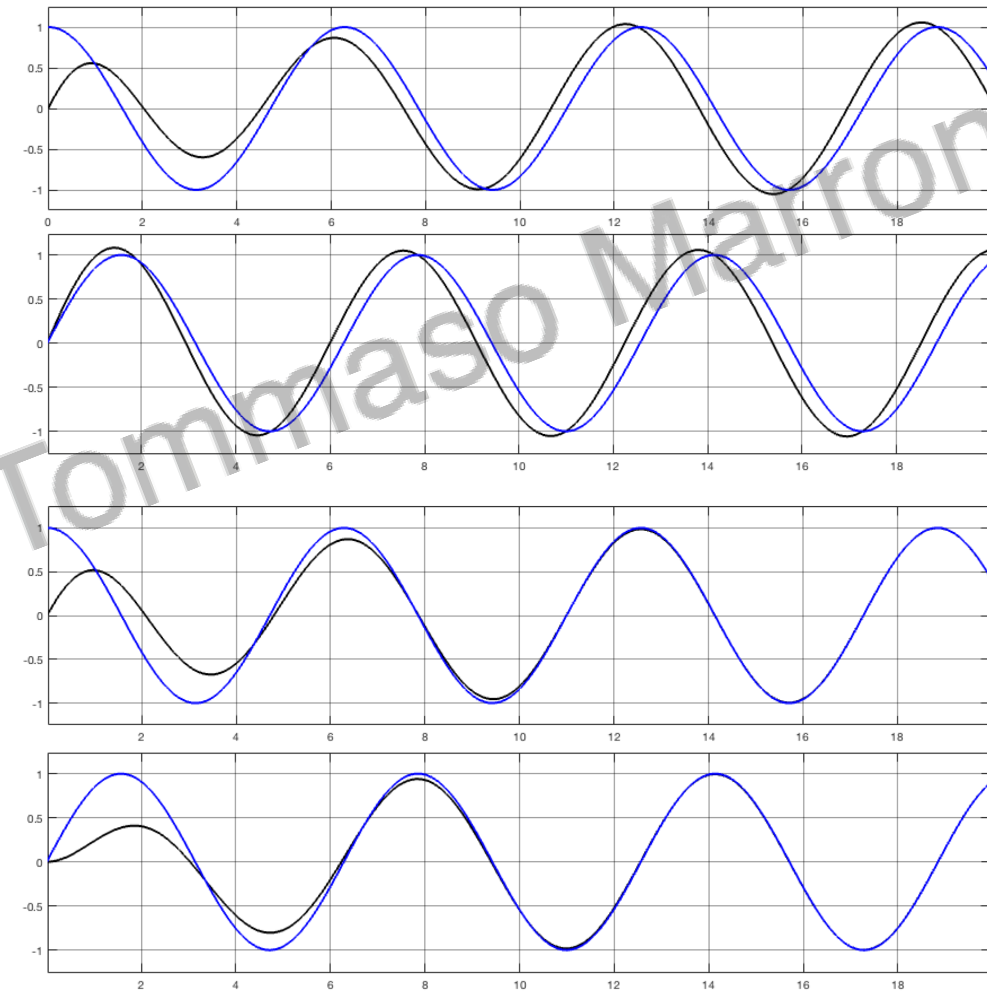
Additional Notes



Deriv. Blocks



No Deriv. Blocks

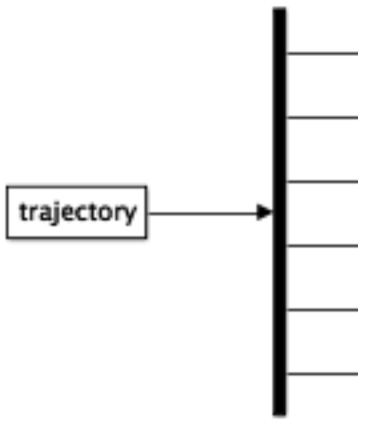


Additional Notes

Issue n° 4 [CST]

```
syms t xd yd Dxd Dyd DDxd DDyd;

%example
xd = sin(t);
yd = cos(t);

Dxd = diff(xd, t);
Dyd = diff(yd, t);
DDxd = diff(Dxd, t);
DDyd = diff(Dyd, t);

t = (0:Ts:T)';

xd = double(subs(xd));
yd = double(subs(yd));
Dxd = double(subs(Dxd));
Dyd = double(subs(Dyd));
DDxd = double(subs(DDxd));
DDyd = double(subs(DDyd));

trajectory.time = t;
trajectory.signals.values = [xd, yd, Dxd, Dyd, DDxd, DDyd];
trajectory.signals.dimensions = 6;
```

- Solution: compute derivatives mathematically using symbolic variables.



- N.B: we have noticed that, in general, using the derivative blocks worsens the performances of all the controllers (also Linear Approximation and RAS)

Conclusion

- The goal of this project was to show the efficiency of the previous controllers through simulations.

- Controllers are able to follow the chosen trajectories.
  - The only exception is linear approximation controller which has some limitations.

- For a more accurate study of the mathematical properties and stability proofs, see [1] and [2].

References

[1] Luigi Freda, Giuseppe Oriolo, *Vision-based interception of a moving target with a nonholonomic mobile robot*, Elsevier, Robotics and Autonomous Systems 55 (2007) 419-432.

[2] Giuseppe Oriolo, Alessandro De Luca, *WMR Control Via Dynamic Feedback Linearization: Design, Implementation, and Experimental Validation*, IEEE Transactions on Control Systems Technology, Vol. 10, No. 6, 835-852, November 2002.

[3] Bruno Siciliano, Lorenzo Sciavicco, Luigi Villani, Giuseppe Oriolo, *Robotics: Modelling, Planning and Control*, Springer, 3rd Edition, 2009.